SHOCK WAVES IN SOLIDS

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by L. P. Orlenko and K. P. Stanyukovich

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SHOCK   WAVES   IN   SOLIDS

The propagation of shock waves in solids under
high pressures considering the compressibility of the
material is examined in this article. The motion of
the material is studied in the zone of an incident shock
wave and the reflected wave zone, where the body has
finite dimensions. The solution is obtained in the
form of finite formulas.
The motion of the incident shock wave is investi-
gated by means of a special solution of differential
equations of motion. The wave reflected from a free
surface is investigated by a general solution of dif-
ferential equations of motion. The results obtained
make it possible to determine all of the parameters of
both incident and reflected waves.
By utilizing the theoretical results obtained,
a dynamic diagram of deformation stress can be derived
with the appropriate experiments.

To solve the dynamic problems arising when
detonation or shock charges are applied to a solid

it is necessary to know the dynamic deformation-stress
curve.   An experimental determination of such diag-
rams presents considerable difficulties. It is known
that when an instantaneous pressure is applied to a
solid, which then falls in the course of time, elastic,
plastic or shock waves can be propagated in the solid,
depending on the magnitude of the applied pressure and
the properties of the solid (see work (1)).

In the present work the solution of the dynamic
problem at high detonation charges is analyzed, where
shock waves are propagated in a solid medium. In this
area of great dynamic pressures, where the solid medium
behaves like a compressible fluid (1), the dynamic
deformation-stress volume diagram is only qualitatively
known, i.e. it can be given, for example, by the equa-
tion having the following form: $\sigma - \sigma_0 = -A(\varepsilon - \varepsilon_0)^{-3}$,
where A, σ_0, ε_0 are unknown constants. In such an
approximation of the $\sigma - \varepsilon$ curve,   one can obtain in
the form of finite formulas the laws of velocity and
pressure change in both the incident shock wave and the
reflected wave (were the body, for instance, to be a
plate of finite thickness). It is known that if impact
loading is applied at one side of a finite body (a
plate), at the rear a part of the material will be split
off with great velocity. Using this phenomenon, it is
possible by means of the solution obtained in this work
to find the unknown constants A, σ_0 and ε_0, the dynamic
deformation-stress volume curve with compression will
be determined by this.

We will examine the propagation of a shock wave
in a solid, where a great instantaneous pressure which
subsequently decreases with time is applied to the lat-
ter.

It is known that the "deformation-stress" com-
pression curve for solids have the form shown in Fig. 1
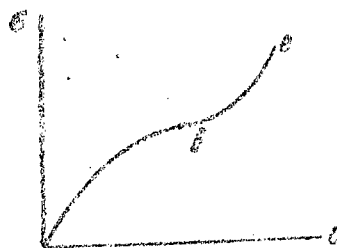(see work (1)).

Fig. 1

If the pressure in the wave exceeds the pressure

corresponding to the discontinuity b of the σ - ε curve, shock waves will be propagated in the solid. It is apparent that under these pressures the solid may be regarded as a quasifluid (1).

It is known that the basic equations of one-dimensional motion have the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0. \qquad \frac{\partial u}{\partial h} = \frac{\partial v}{\partial t} . \qquad (1)$$

where u - is the particle velocity; v - is specific volume; t - is time; p - pressure; h - is the Lagrangian coordinate.

If one introduces the terms $\sigma = -p$, $\varepsilon = \frac{v - v_0}{v_0} = \frac{\rho_0}{\rho} - 1$, $\qquad (2)$

then we obtain

$$\frac{\partial u}{\partial t} = \frac{\partial \sigma}{\partial h} = \frac{d\sigma}{d\varepsilon} \frac{\partial \varepsilon}{\partial h} . \qquad \frac{\partial u}{\partial h} = v_0 \frac{\partial \varepsilon}{\partial t} . \qquad (3)$$

The equation of state (curve b - e in Fig. 1) is approximated by the relation having the following form:

$$\sigma - \sigma_0 = - A(\varepsilon - \varepsilon_0)^{-3}, \qquad (4)$$

where σ_0, A and ε_0 are constants.

We then obtain

$$\frac{d\sigma}{d\varepsilon} = \frac{3A}{(\varepsilon - \varepsilon_0)^{+4}} . \qquad (5)$$

The velocity of sound in this case is determined by the expression

$$c = \sqrt{\frac{dp}{d\rho}} = (\varepsilon + 1)\sqrt{\frac{d\sigma}{\rho_0 d\varepsilon}} = \frac{\varepsilon + 1}{(\varepsilon - \varepsilon_0)^2}\sqrt{\frac{3A}{\rho_0}} =$$

$$= \frac{\sqrt{3A\rho_0}}{\rho\left[\frac{\rho_0}{\rho} - (1 + \varepsilon_0)\right]^2} . \qquad (6)$$

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The velocity of propagation of disturbances from particle to particle will be equal to

$$w = \sqrt{\frac{d\sigma}{\rho_0 \, d\varepsilon}} = \frac{c}{\varepsilon + 1} = \frac{\rho}{\rho_0} \qquad c = \frac{1}{(\varepsilon - \varepsilon_0)^2} \sqrt{\frac{3A}{\rho_0}} =$$

$$= \sqrt{\frac{3A}{\rho_0}} \frac{1}{\left[\frac{\rho_0}{\rho} - (1 + \varepsilon_0)\right]^2}. \qquad (7)$$

We write equation (3) in the form

$$\frac{\partial u}{\partial t} = \frac{d\sigma}{d\varepsilon} \frac{\partial \varepsilon}{\partial h} = \frac{3A}{(\varepsilon - \varepsilon_0)^4} \frac{\partial(\varepsilon - \varepsilon_0)}{\partial h}.$$

$$\frac{\partial u}{\partial h} = v_0 \frac{\partial(\varepsilon - \varepsilon_0)}{\partial t}. \qquad (8)$$

We will find particular solutions for this system of equations. We will assume for this purpose, as usual, that $\varepsilon = \varepsilon(u)$, then we will arrive at the equations

$$\frac{\partial u}{\partial t} = \frac{d\sigma}{du} \frac{\partial u}{\partial h}, \qquad \frac{\partial u}{\partial h} = v_0 \frac{d\varepsilon}{du} \frac{\partial u}{\partial t}, ,$$

whence we obtain

$$\frac{du}{d\sigma} = v_0 \frac{d\varepsilon}{du} \qquad\qquad \text{or}$$

$$du = \sqrt{v_0 \, d\varepsilon \, d\sigma} = \sqrt{v_0 \frac{d\sigma}{d\varepsilon}} \, d\varepsilon = w \, d\varepsilon. \qquad (9)$$

$$\frac{\partial u}{\partial t} = \rho_0 \frac{\partial u}{\partial h} \frac{du}{d\varepsilon} = \frac{\partial u}{\partial h} \sqrt{\rho_0 \frac{d\sigma}{d\varepsilon}}. \qquad (10)$$

The solution of equation (10) has the form

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$$h = \sqrt{\rho_0 \frac{d\sigma}{d\varepsilon}}\; t + F(u) = \rho_0 \, wt + F(u). \tag{11}$$

In the equation of state (4) the specific solution may be written in the form

$$
\left.
\begin{aligned}
u &= \pm \sqrt{\frac{3A}{\rho_0}\frac{1}{\varepsilon - \varepsilon_0}} + const \\
h &= \pm \sqrt{\frac{3A}{\rho_0}\frac{\rho_0 t}{(\varepsilon - \varepsilon_0)^3}} + F(u)
\end{aligned}
\right\} \tag{12}
$$

We will now seek the general solutions of the system of equations (9). Having inverted the dependent and independent variables, we arrive at the equations:

$$v_0 \frac{\partial h}{\partial \varepsilon} = w^2 \frac{\partial t}{\partial u}, \qquad \frac{\partial t}{\partial \varepsilon} = v_0 \frac{\partial h}{\partial u}. \tag{13}$$

We find from this that

$$v_0 \frac{\partial^2 h}{\partial \varepsilon \, \partial u} = \frac{\partial^2 t}{\partial \varepsilon^2} = w^2 \frac{\partial^2 t}{\partial u^2}. \tag{14}$$

In the equation of state (4) we will have

$$\frac{\partial^2 t}{\partial (\varepsilon - \varepsilon_0)^2} = \frac{3A}{\rho_0 (\varepsilon - \varepsilon_0)^4} \frac{\partial^2 t}{\partial u^2}. \tag{15}$$

We will introduce the new variable $z = \frac{\alpha}{\varepsilon - \varepsilon_0}$, then $\varepsilon - \varepsilon_0 = A\left(\frac{z}{\alpha}\right)^3$, and equation (15) assumes the form

$$\frac{\partial^2 t}{\partial z^2} + \frac{2}{z}\frac{\partial t}{\partial z} = \frac{3A}{\rho_0 \alpha^2}\frac{\partial^2 t}{\partial u^2}. \tag{16}$$

Assuming that $\alpha^2 = \frac{3A}{\rho_0}$, then

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$$z = \sqrt{\frac{3A}{\rho_0}} \frac{1}{\varepsilon - \varepsilon_0} = w(\varepsilon - \varepsilon_0) = \frac{\varepsilon - \varepsilon_0}{\varepsilon + 1} \qquad c = \frac{aw}{z} \ .$$

$$w = \frac{z^2}{a} \ ,$$

equation (16) assumes the form

$$\frac{\partial^2(zf)}{\partial z^2} = \frac{\partial^2(zf)}{\partial u^2} \ . \tag{17}$$

The solution of this equation:

$$t = \frac{\Phi_1(u-z) + \Phi_2(u+z)}{z} = \frac{F_1'(u-z) + F_2'(u+z)}{z} \ . \tag{18}$$

where Φ_1, Φ_2, F_1 and F_2 are arbitrary functions (the prime designates differentiation according to the arguments).

We find from the 2nd equation (13) that

$$\frac{\partial t}{\partial \varepsilon} = -\frac{z^2}{a} \frac{\partial t}{\partial z} = + \left[\frac{F_1'' - F_2''}{z} + \frac{F_1' + F_2'}{z^2} \right] \frac{z^2}{a} = \frac{1}{\rho_0} \frac{\partial h}{\partial u} \ .$$

whence it follows that

$$h = \frac{\rho_0}{a} \left[z(F_1' - F_2') + (F_1 + F_2) \right] . \tag{19}$$

If the function $\varphi = \varphi(\varepsilon, u)$ is introduced which satisfies the second equation (13), then

$$t = \frac{\partial \psi}{\partial u} \ . \qquad h = \rho_0 \frac{\partial \psi}{\partial \varepsilon} = -\frac{\rho_0 z^2}{a} \frac{\partial \psi}{\partial z} \ . \tag{20}$$

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Whereupon the solution of (18) assumes the form

$$\psi = \frac{f_1(u-z) + f_2(u+z)}{z},$$ (21)

there f_1 and f_2 are new arbitrary functions,

$$t = \frac{f_1 + f_2}{z} \qquad k = \frac{p_0}{a}\left[(f_1' - f_2')z + f_1 + f_2.\right].$$ (22)

The special solution of (11) assumes the form

$$\frac{p_0 z^2}{a}\frac{\partial \psi}{\partial z} = z\sqrt{-\frac{p_0}{a}\frac{d\sigma}{dz}}\frac{\partial \psi}{\partial u} + F(u).$$ (23)

Whereupon

$$du = -\frac{1}{z}\sqrt{-\frac{d\sigma}{p_0\,dz}}\,dz.$$ (24)

We multiply (23) by du, then

$$z\frac{\partial \psi}{\partial z}\sqrt{-\frac{p_0}{a}\frac{d\sigma}{dz}} + z\sqrt{-\frac{p_0}{a}\frac{d\sigma}{dz}}\frac{\partial \psi}{\partial u}du = F(u)\,du.$$

Whence

$$z\sqrt{-\frac{p_0}{a}\frac{d\sigma}{dz}}\,d\psi = F(u)\,du \text{ or } d\psi =$$
$$= \Phi(u)\,du,$$ (25)

there

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$$\Phi(u) = \frac{F(u)}{z\sqrt{-\dfrac{\rho_0}{a}\dfrac{d\sigma}{dz}}} = \frac{F(u)}{\rho_0\dfrac{\varepsilon-\varepsilon_0}{a}\,z\sqrt{\dfrac{1}{\rho_0}\dfrac{d\sigma}{d\varepsilon}}} =$$

$$= \frac{F(u)}{z_0\sqrt{\dfrac{1}{\rho_0}\dfrac{d\sigma}{dz}}}$$

or

$$\Phi(u) = \frac{F(u)}{\rho_0 x'},$$

Hence, along the characteristics in plane (u, z) which are determined by the special solution

$$\psi = \psi(u) = \psi(z). \tag{26}$$

If $F(u) = 0$, then $\psi = $ const., since without limiting the general representation it may always be assumed that const.$= 0$, hence if $F(u)=0$, along the characteristic $\psi = 0$ (27). In the equation of state (4), we have

$$u = \pm z + \text{const.}$$

$$h = \pm \frac{\rho_0 + tz^2}{a} + F(z) = \rho_0 ut + F(z). \tag{28}$$

If, for example, we determine ψ along the characteristic $u + z = $ const., then in this individual case, where $F(z) = 0$, $f_2 = 0$.

If ψ is determined along the characteristic $u - z = $ const., then when $F(z) = 0$, $f_1 = 0$.

We will look at the solution of a concrete example.

Let us assume at the moment of time $t = 0$ in cross-section $h = 0$ there is a pressure $p = p_R$ applied, where

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p_H is greater than σ_δ, where σ_δ is the pressure corresponding to the discontinuity point in curve $\sigma - \epsilon$ (Fig.1) which then is found according to the law

$$\frac{p - p_H}{p_H - p_0} = \left(\frac{\tau}{t + \tau}\right)^{3n},\qquad (29)$$

where p_0, τ and n are constants.

It is known from detonation theory that it is possible to assume $p_0 = 0$, $n = 1$ and $\tau = \frac{1}{D}$, where 1 is the height of the loading and D is the velocity of the detonation.

We find the law of motion of the medium in the passing wave. The motion of the medium in the wave will be subject to the specific solution (28) where it is necessary to determine $F(z)$:

$$\left.\begin{array}{c} u - z = \text{const} \\[2mm] h = \dfrac{p_0 l z^2}{a} + F(z) \end{array}\right\} .\qquad (A)$$

We will determine the constant in the first equation of the system (A).

When $u = u_H$, $z = z_H$, inasmuch as at the wave front

$$\sigma_H = \frac{u^2{}_H \rho_0}{\epsilon_H}, \qquad \text{then const.} = -z_0 = \sqrt{\frac{\sigma_H \cdot \epsilon_H}{\rho_H} \cdot \frac{a}{\epsilon_H - \epsilon_\psi}}$$

Using equation (4), (29) can be written in the form

$$\frac{z}{z_H} = \left(\frac{\tau}{t + \tau}\right)^n .\qquad (30)$$

The first equation (A) now assumes the form: $u = z - z_0$.

$$(31)$$

For the initial moment

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$$u_H = z_H - z_0. \tag{32}$$

If it is assumed in equation (A) that h = 0, and the time t is expressed from (30), then we determine (z):

$$F(z) = -\frac{\rho_0 z^2 \tau}{a}\left[\left(\frac{z_H}{z}\right)^{\frac{1}{n}} - 1\right]. \tag{33}$$

System (A) is finally written in the form

$$u = z - z_0,$$

$$h = \frac{\rho_0 z^2}{a}\left\{t - \tau\left[\left(\frac{z_H}{z}\right)^{\frac{1}{n}} - 1\right]\right\}. \tag{34}$$

If n = 1, we derive the quadratic equation relative to z:

$$z^2 - \frac{z_H \tau}{1 + \tau}z - \frac{ah}{\rho_0(1 + \tau)} = 0.$$

The solution of this equation will be

$$z = u + z_0 = \frac{z_H \tau \pm \sqrt{z_H^2 \tau^2 + 4 v_0 a h (1 + \tau)}}{2(1 + \tau)}. \tag{35}$$

We find the law of motion of the shock front = h(t):

$$D_y = \frac{dh}{\rho_0 dt} = \frac{u + c + c_0}{2} = \frac{z - z_0 + c + c_0}{2}, \tag{36}$$

where

D_y is the velocity of the shock wave,

c_0 is the velocity of sound in metal.

We express the local velocity of sound c by z:

$$c = \frac{\varepsilon + 1}{\varepsilon - \varepsilon_0} z = \frac{(\varepsilon_0 + 1) z^2 + az}{a} = z + \frac{\varepsilon_0 + 1}{a} z^2. \qquad (37)$$

Thus, the velocity of wave D_y will be equal to

$$D_y = z + \frac{c_0 - z_0}{2} + \frac{\varepsilon_0 + 1}{2a} z^2. \qquad (38)$$

By means of equation (35) we eliminate z:

$$\frac{dh}{\rho_0 \, dt} = \frac{c_0 - z_0}{2} + \frac{(\varepsilon_0 + 1) h}{2\rho_0(t + \tau)} +$$

$$+ \left[1 + \frac{\varepsilon_0 + 1}{2a} \cdot \frac{z_H \tau}{t + \tau} \right] \frac{z_H \tau + \sqrt{z_H^2 \tau^2 + 4v_0 ah(t + \tau)}}{2(t + \tau)}. \qquad (39)$$

Integrating this equation which when h = 0, t = 0, we find the law of propagation of the shock wave:

$$h = h(t). \qquad (40)$$

If the body has finite thickness d, then when the wave reaches the back side of the solid it will be reflected from the free surface as a rarefaction (expansion /Russian: discharge 7) wave, moving toward the shock wave. The motion of the medium in the rarefaction wave is determined by the general solution (22). The arbitrary functions in the general solution are determined primarily from the condition of the connection between the reflected and incident waves; secondarily, by the fact that when $t \geq T$; $h = \rho_0 d$, $\tau = 0$, where d is the thickness of the plate. The first condition implies that on the line $u = z = z_0$ takes place according to (25) and (33):

$$\psi = \int \Phi(u) du = \frac{u}{\rho_0} \int \frac{F(z)}{z^2} \, dz = \tau \int \left[\left(\frac{z_H}{z} \right)^{\frac{1}{n}} - 1 \right] dz =$$

$$= \tau z \left[-1 + \frac{n}{n-1} \left(\frac{z_H}{z} \right)^{\frac{1}{n}} \right].$$

If n = 1, then $\psi = \tau z \left[-1 + \dfrac{z_\kappa}{z} \ln \dfrac{z}{z_\kappa^*} \right] = \dfrac{f_1 + f_2}{z}$

or

$$-\tau z^2 + \tau z_\kappa z \ln \dfrac{z}{z_\kappa^*} = f_1 + f_2, \qquad (41)$$

where $f_1 = f_1(2z - z_0)$ and $f_2 = f_2(-z_0)$,

z_κ^* is the integration constant.

The second condition provides that when $h = \rho_0 d$ the condition

$$ad = f_1 + f_2 + (f_2' - f_1') z_d, \qquad (42)$$

is fulfilled, where

$$z_d = a \sqrt{\dfrac{\sigma_0}{A}}$$

Thus, by means of (41) and (42) the parameters of the reflected wave can be determined. The found solution is complicated for investigation, hence we will find a less precise, although considerably simpler solution of the problem. It is known from detonation theory that the pressure on the surface of the plate falls according to the law

$$p = p_\kappa \left(\dfrac{l}{Dt} \right)^2, \qquad (43)$$

where

$$p_\kappa = \dfrac{16}{27} \rho_0 D^2 \qquad (44)$$

The velocity of the shock wave D_y is determined by the formula

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$$D_y = \frac{u + c + c_a}{2} = \frac{dx}{dt}. \tag{45}$$

We take the equation of state in the form

$$p = A_1 \rho^k - B_1. \tag{46}$$

where A_1, B_1 and k are constants.

The motion of the shock wave will be determined by special solutions of the system of equations (1) written in Euler's form:

$$c = c_a + \frac{k-1}{2} u. \tag{47}$$

$$x = (u + c)t + F(u). \tag{48}$$

The velocity of the shock wave D_y is determined by the expression

$$D_y = \frac{dx}{dt} = c_a + \frac{k+1}{4} u. \tag{49}$$

Function $F(u)$ is determined by the condition that when $x = 0$ the pressure varies according to the law (43):

$$p = p_\varkappa \left(\frac{l}{Dt} \right)^3$$

whence follows that

$$\frac{Dt}{l} = \left(\frac{p_\varkappa}{p} \right)^{\frac{1}{3}} = \frac{\overline{c}_\mu}{c}, \tag{50}$$

where $\overline{c}_\mu$ is the initial speed of sound in the plate where $p = p_\varkappa$,

$$\overline{c}_N = \sqrt{\frac{dp}{d\rho}} = \sqrt{kA_1} \left(\frac{p_\varkappa + B_1}{A_1} \right)^{\frac{k-1}{2k}}. \tag{51}$$

By means of (47) equation (50) assumes the form

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$$t = \frac{l}{D} \left(\frac{c_a + \frac{k-1}{2} u}{\overline{c}_{\scriptscriptstyle H}} \right)^{-1} . \tag{52}$$

When x = 0 we determine F(u) according to (48) and (52):

$$F(u) = - \left(c_a + \frac{k+1}{2} u \right) \left(c_a + \frac{k-1}{2} u \right)^{-1} \frac{l}{D \overline{c}_{\scriptscriptstyle H}} .$$

Equation (48) now has the form

$$x = \left(C_a + \frac{k+1}{2} u \right) \left[t - \left(\frac{c_a + \frac{k-1}{2} u}{\overline{c}_{\scriptscriptstyle H}} \right)^{-1} \frac{l}{D} \right] . \tag{53}$$

When k = 3 this expression is written in the form

$$x = (c_a + 2u) \left(t - \frac{\overline{c}_{\scriptscriptstyle H}}{u + c_a} \frac{l}{D} \right) . \tag{54}$$

We convert this expression to the form

$$x (u + c_a) \frac{D}{l} = (c_a + 2u) \left[\frac{Dt}{l} (u + c_a) - \overline{c}_{\scriptscriptstyle H} \right] , \tag{55}$$

or

$$2 \frac{Dt}{l} u^2 + u \left[\frac{Dt}{l} 3 c_a - x \frac{D}{l} - 2 \overline{c}_{\scriptscriptstyle H} \right] + \frac{Dt}{l} c_a^2 - $$
$$- \overline{c}_{\scriptscriptstyle H} c_a - x c_a \frac{D}{l} = 0 .$$

Inasmuch as $u \ll c_0$, neglecting the term with u^2, we find that

$$u = \frac{\overline{c}_H + x\,\dfrac{D}{l} - \dfrac{Dt}{l}\,c_a}{\dfrac{Dt}{l}\,3 - \dfrac{x}{c_a}\dfrac{D}{l} - 2\dfrac{\overline{c}_H}{c_a}}\,;\tag{56}$$

When $k = 3$ equation (49) assumes the form

$$D_y = \frac{dx}{dt} = u + c_a = \frac{c_a\left[2\dfrac{Dt}{l} - \dfrac{c_H}{c_a}\right]}{\dfrac{Dt}{l}\,3 - \dfrac{x}{c_a}\dfrac{D}{l} - 2\dfrac{c_H}{c_a}}\,.\tag{57}$$

We assume $\dfrac{Dt}{l} = \tau + \dfrac{\overline{c}_H}{2c_a}$, $\qquad \xi = \dfrac{\overline{c}_H}{2D} + \dfrac{x}{l}$,

we then get

$$\frac{d\xi}{d\tau} = \frac{c_a}{D}\,\frac{2\tau}{3\tau - \dfrac{D}{c_a}\cdot\xi}\,.\tag{58}$$

We will designate the relation $\dfrac{\xi}{\tau} = z$, then

$$\frac{dz}{d\tau}\,\tau = \frac{c_a}{D}\,\frac{2}{3 - \dfrac{D}{c_a}z} - z$$

or

$$d\ln\tau = \frac{Dz}{\dfrac{c_a}{D}\,\dfrac{2}{3 - D/c_a\cdot z} - z} = \frac{\left(3 - \dfrac{D}{c_a}z\right)dz}{\dfrac{2c_a}{D} - 3z + \dfrac{D}{c_a}z^2}$$

After integration we obtain

$$\tau = A \left| \frac{z - 2\dfrac{c_a}{D}}{\left(z - \dfrac{c_a}{D}\right)^2} \right| \qquad , \qquad (60)$$

where A is the integration constant.
     The constant A is determined from the condition that when x = 0,

$$t = \frac{l}{D} \quad \text{or when } \tau = 1 \quad \tau = 1 - \frac{\overline{c}_{\scriptscriptstyle K}}{2c_a}$$

$$t = \frac{\overline{c}_{\scriptscriptstyle H}}{2D}, \qquad\qquad\qquad z = \frac{\overline{c}_{\scriptscriptstyle H} c_a}{D(2c_a - \overline{c}_{\scriptscriptstyle H})}.$$

The constant A is equal to

$$A = -\frac{2\overline{c}_{\scriptscriptstyle H}}{D} \; \frac{\left(1 - \dfrac{c_a}{\overline{c}_{\scriptscriptstyle H}}\right)^2}{4\dfrac{c_a}{\overline{c}_{\scriptscriptstyle H}} + 1}. \qquad\qquad (60)$$

Equation (59) can now be written in the form

$$\tau = -\frac{2\overline{c}_{\scriptscriptstyle H}}{D} \; \frac{\left(1 - \dfrac{c_a}{\overline{c}_{\scriptscriptstyle H}}\right)^2}{4\dfrac{c_a}{\overline{c}_{\scriptscriptstyle H}} + 1} \; \frac{z - 2\dfrac{c_a}{D}}{\left(z - \dfrac{c_a}{D}\right)^2}. \qquad (61)$$

Let us turn to the new variables according to equation (58)

$$x^2 + xl\left(2\frac{\overline{c}_{\scriptscriptstyle H}}{D} - \frac{2tc_a}{l} - A\right) + \left(\frac{c_{\scriptscriptstyle H}}{D} - \frac{tc_a}{l}\right)^2 l^2 +$$

$$+ A\left(\frac{3}{2}\frac{\overline{c}_{\scriptscriptstyle H}}{D} - 2\frac{tc_a}{l}\right)l^2 = 0. \qquad (62)$$

From this we determine x = x(t) — the law of motion of the shock wave:

$$x = -\frac{l}{2}\left(2\,\frac{\overline{c_R}}{D} - \frac{2lc_a}{l} - A\right) \pm$$

$$\pm \sqrt{\frac{l^2}{4}\left(\frac{2\overline{c_R}}{D} - \frac{2lc_a}{l} - A\right)^2 - B} \quad , \qquad (63)$$

where

$$B = \left(\frac{\overline{c_M}}{D} - \frac{lc_a}{l}\right)^2 l^2 + A\left(\frac{3}{2}\,\frac{\overline{c_R}}{D} - 2\,\frac{lc_a}{l}\right)l^2. \qquad (64)$$

The obtained relations can be used to determine pressure in the passing waves.

To determine the pressure in the passing wave at a given distance x = h one must determine t_h according to formula (63) when x = h, then according to (56) determine u_h, in accordance with which c_h is determined with (47). The pressure is determined by means of the equation of state (46).

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